Exercise 1

A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume V of water remaining in the tank (in gallons) after t minutes.

t (min)	5	10	15	20	25	30
V (gal)	694	444	250	111	28	0

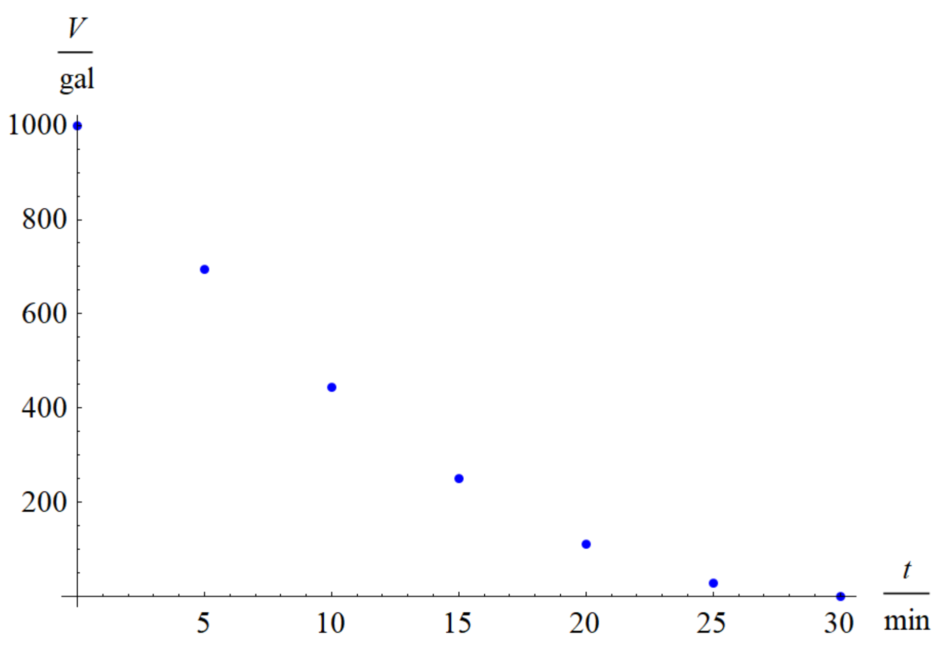
(a) If P is the point $(15, 250)$ on the graph of V, find the slopes of the secant lines PQ when Q is the point on the graph with $t = 5$, 10, 20, 25, and 30.

(b) Estimate the slope of the tangent line at P by averaging the slopes of two secant lines.

(c) Use a graph of the function to estimate the slope of the tangent line at P. (This slope represents the rate at which the water is flowing from the tank after 15 minutes.)

Solution

Below is a plot of the points listed in the table: $(0, 1000)$, $(5, 694)$, $(10, 444)$, $(15, 250)$, $(20, 111)$, $(25, 28)$, and $(30, 0)$.



Part (a)

Use the slope formula for the two points on each secant line.

$$(15, 250) \text{ and } (5, 694): \qquad m = \frac{694 - 250}{5 - 15} = -44.4$$

$$(15, 250) \text{ and } (10, 444): \qquad m = \frac{444 - 250}{10 - 15} = -38.8$$

$$(15, 250) \text{ and } (20, 111): \qquad m = \frac{111 - 250}{20 - 15} = -27.8$$

$$(15, 250) \text{ and } (25, 28): \qquad m = \frac{28 - 250}{25 - 15} = -22.2$$

$$(15, 250) \text{ and } (30, 0): \qquad m = \frac{0 - 250}{30 - 15} \approx -16.7$$

The equations for the secant lines are obtained by using the point-slope formula.

$$V - 250 = -44.4(t - 15)$$

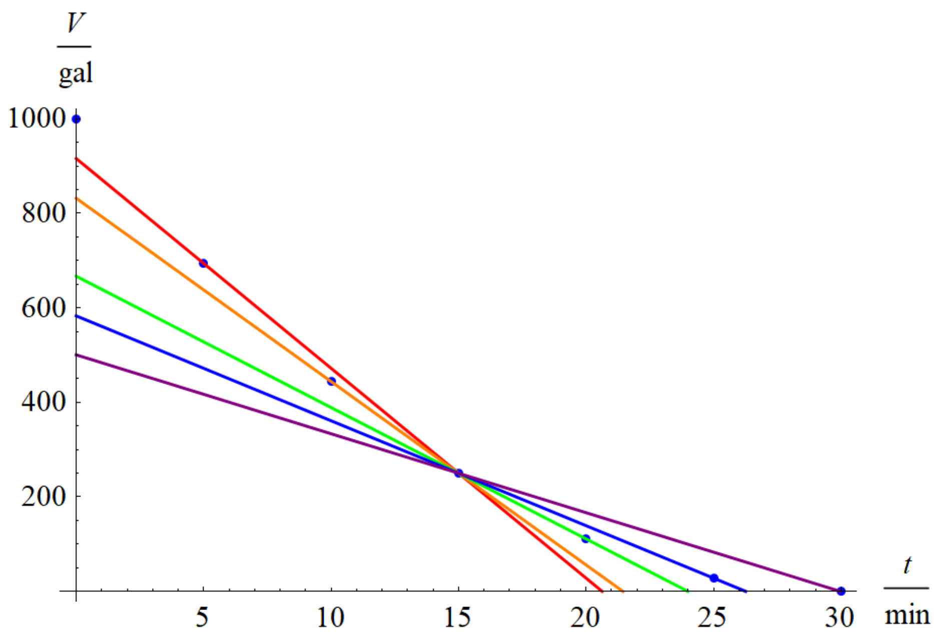
$$V - 250 = -38.8(t - 15)$$

$$V - 250 = -27.8(t - 15)$$

$$V - 250 = -22.2(t - 15)$$

$$V - 250 \approx -16.7(t - 15)$$

Plot these lines versus t with the scatterplot.



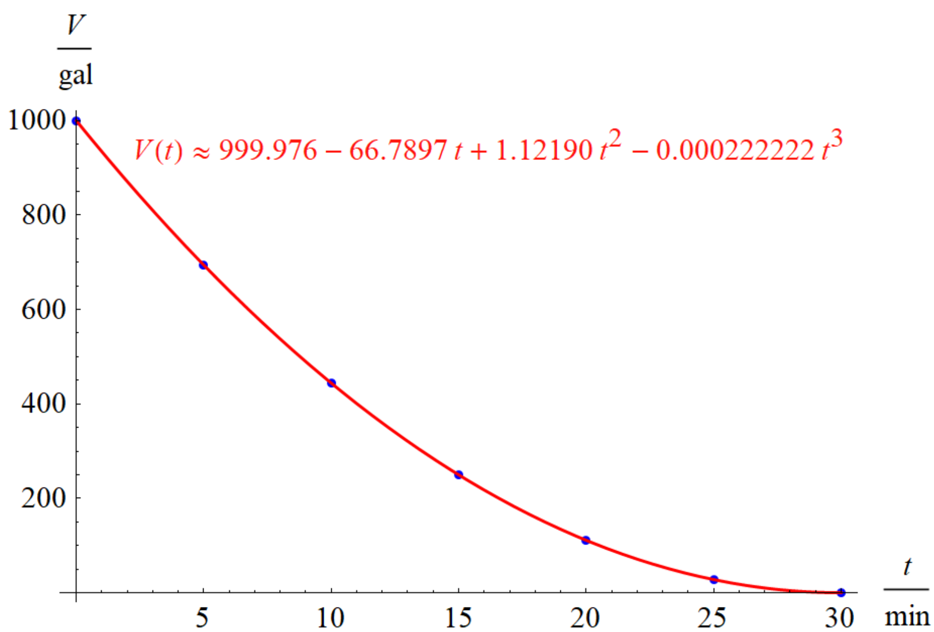
Part (b)

Use the slopes of the secant lines going through the points closest to $(15, 250)$, $(10, 444)$ and $(20, 111)$, to get the most accurate slope of the tangent line.

$$m_{\text{tangent}} = \frac{-38.8 + (-27.8)}{2} = -33.3$$

Part (c)

Use Mathematica's Fit function to obtain the optimal cubic curve going through the data.



The slope of the tangent line at $t = 15$ is about -33.2825, which is consistent with the result in part (b).

$$\begin{aligned}
\frac{dV}{dt} &\approx -66.7897 + 2(1.12190)t - 3(0.000222222)t^2 \\
\left.\frac{dV}{dt}\right|_{t=15} &\approx -66.7897 + 2(1.12190)(15) - 3(0.000222222)(15)^2 \\
&\approx -33.2825
\end{aligned}$$